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Design and Analysis of Halo orbits around L1 Libration point for Sun-Earth system

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Abstract— In the last few years, the interest concerning the libration points for space applications has risen within scientific community. This is because the libration points are the natural equilibrium solutions of the restricted three body problem. In this paper, design of halo orbit around L1 point for the sun-earth system is done by known non-linear differential equations by their initial conditions by differential correction and differential evolution methods.

Index Terms— (Restricted three body problem, lagrangian points, Halo orbits, Differential correction, Differential evolution)

1 Introduction

The problem of three body systems is to determine the motion of three bodies under the influence of their mutual gravitational forces. If the mass of one of the bodies is a so small compared to the other two that it cannot influence their motion, then such systems are called restricted three body systems. This is the case of a spacecraft moving in the gravitational fields of two massive bodies like the Earth and the Sun or the Earth and the Moon. The problem of describing the motion of the smaller third body in such a system is called restricted three body problem. The two larger bodies are referred to as primaries. Further if system of the two larger bodies undergoes a circular orbital motion, then the problem is called circular restricted three body problem or CRTBP in short
Brief analysis of the circular restricted three-body problem (CRTBP) model has been done here, as we used in this study. In this model, the third body (spacecraft), assumed very small
in comparison to the two primary bodies. Around a commaon center of mass known as barycenter, the two primaries are assumed to rotate in circular orbits at barycenter. The origin of the coordinate frame is fixed and rotates with the rotation of primaries. As shown in the fig1.

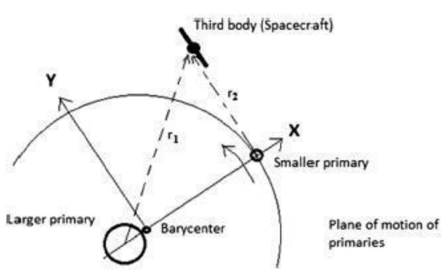

Fig 1: CRTBP

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1.2 Lagrange points

In solar system the locations where the gravitational pull from one massive body outweighs the pull of another massive body is said to be Lagrange points. Just as in earth and moon or sun and earth, creating points where satellite remains in orbit with less effort. As seen in Figure 2 below there are five different points. For the Earth-Sun system, which this report will focus on, the first two points **L1** and **L2** are located on the opposite sides of the Earth. **L3** point lies on the line defined by the two large masses, beyond the larger of the two.

The **L4** and **L5** points lie at the third corners of the two equilateral triangles in the plane of orbit whose common base is the line between the centers of the two masses, such that the point lies behind (L_5) or ahead (L_4) of the smaller mass with regard to its orbit around the larger mass.

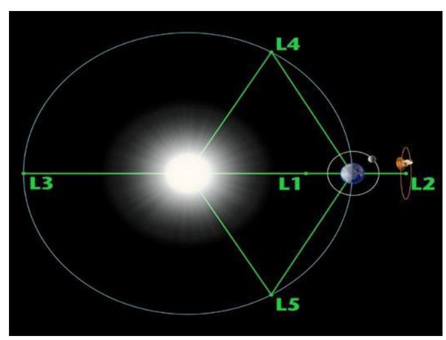

Fig 2: Lagrange points for sun-earth system

1.3 Halo orbits

The name Halo orbits were first used in the PhD thesis of Robert W Farquhar in 1968. One of the most frequently studied models in the celestial mechanics is the three-body problem. The complicated interaction between the gravitational pull of the two-planetary bodies, the cariols and centrifugal accelerations on a spacecraft results in halo orbit. The motion resulting from particular initial conditions, which produce periodic, three-dimensional 'halo' orbits. A halo orbit is a periodic, three-dimensional orbit near the L_1, L_2or L_3 Lagrange points in the three-body problem of orbital mechanics. Although the Lagrange point is just a point in empty space, its peculiar characteris-

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tic is that it can be orbited. Halo orbits exist in any three-body system, e.g. the Sun–Earth system and the Earth– Moon system. Continuous "families" of both Northern and Southern halo orbits exist at each Lagrange point. Because halo orbits tend to be unstable, station keeping is required to keep a satellite on the orbit

2. EQUATIONS OF MOTION

The three body problem involves the two finite masses m1 and m2, assumed to be a point mass, moving around their common center, each under the gravitational influence of the other. A rotating coordinate system, with origin at the barycenter is chosen as shown in below figure 3. μ is defined as the mass ratio m2 to sum m1+m2.

The mass ratios of some familiar system are: Earth-Moon (0.012), Sun-Jupiter ($9.5*10^{-4}$) and Sun-Earth ($3*10^{-6}$). The x-y plane is the plane of motion of m1 and m2. A z-axis out of the paper competes the right handed system. The third body, m3, is assumed massless but may traveling all of the three dimension. In this system, it is well known that there are five equilibrium points, or liberation points, where gravitational and centrifugal forces balance each other. Temporarily assuming m2<m1, the points are defined as shown in the figure. Of the collinear points, L3 is defined as being on the far side of the larger mass, L1 is between them, and barycenter, all five points remain in the same position relative to the masses for a given μ. For convenience, non-dimensional units were chosen such that the following quantities are equal to unity: the angular velocity of the rotating frame, the distance between m1 and m2, and the sum of the primary masses, m1+m2.

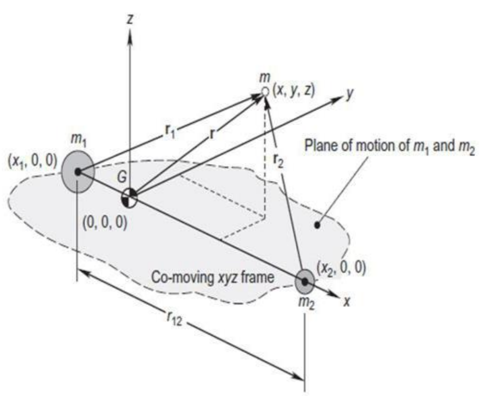

Fig 3: Three body problem

The following are the equations of motion of the third body given Szebehely

$$\ddot{x} - 2\Omega\dot{y} - \Omega^2 x = -\frac{\mu_1}{r_1^3}(x + \pi_2 r_{12}) - \frac{\mu_2}{r_2^3}(x - \pi_1 r_{12})$$

$$\ddot{y} + 2\Omega\dot{x} - \Omega^2 y = -\frac{\mu_1}{r_1^3}y - \frac{\mu_2}{r_2^3}y$$

$$\ddot{z} = -\frac{\mu_1}{r_1^3}z - \frac{\mu_2}{r_2^3}z$$

The equilibrium solutions of the above shown equations of motion lead to five libration points,also known as lagrangain points or lagrange points.some special type of solutions of circular restricted three body problem result in a motion that are periodic and quasi periodic about the lagrange points.this study pertains to design of halo orbit about a collinear libration point between two primaries,Sun and Earth.

3. HALO ORBIT SIMULATION

3.1 Halo orbit simulation by differential correction

The design is based ob DC Scheme as well as DE scheme use the halo orbit characterstics that (i) simple periodic halo orbits pierce the X-Z plane always orthogonally , (ii) again after half the period, the orbit must cross the X_Z plane orthhogonally.
So, the state vectors at t_0 and at half period ($t_{T/2}$) are

$X(t_0) = [\ x_0, 0, z_0, 0, \dot{y}_0, 0]$

$X(t_{T/2}) = [\ x_{T/2}, 0, z_{T/2}, 0, \dot{y}_{T/2}, 0]$

The suitable initial conditions [x_0, z_0,$\dot{y}_0$] are selected such that at half –period x and z velocity components are equal to zero to ensure the orthogonal crossing that results in a halo orbit.

3.2 Differentional Correction procudure

Halo orbits are type of the periodic symmetric orbits that are symmetric about y=0 and pierce that plane twice per orbit. These can be constructed using DC (differential correction) method. The method takes imperfect but close guess of the orbit state X at t0, and integrates that forward in time until the orbit crosses the y-plane, which it does at half an orbit period. Integration is performed using the state transition matrix phi (t0, tf) which is found from the partial derivatives of the state.

$$\phi\ (t, t_0) = \eth X(t)/\eth X(t_0).$$

The state transition matrix can then be propagated forward in time using

$$\dot{\phi}\ (t,\ t_0) = A(t)\phi(t,\ t_0)$$

where A = ðX(t)/ðX(t).
The propagated state at the plane intersection (T/2) is

$\hat{X}(T/2) = [\ x_{T/2}\ \ 0\ z_{T/2}\ \ \dot{x}_{T/2}\ \ \dot{y}_{T/2}\ \ \dot{z}_{T/2}]^T$

In order for the orbit to be periodic and symmetric it has to have the following initial form [4]:

$$X(t_0) = [\ x_0\ \ 0\ \ z_0\ \ 0\ \dot{y}\ 0\ ]^T$$

Based on the state at (T/2) it is necessary to perform some changes to the initial state in order to generate a true halo orbit. The initial states x velocity and z position need to be driven to using deviation in the x position and z velocity which are applied to the initial state in an iterative process.

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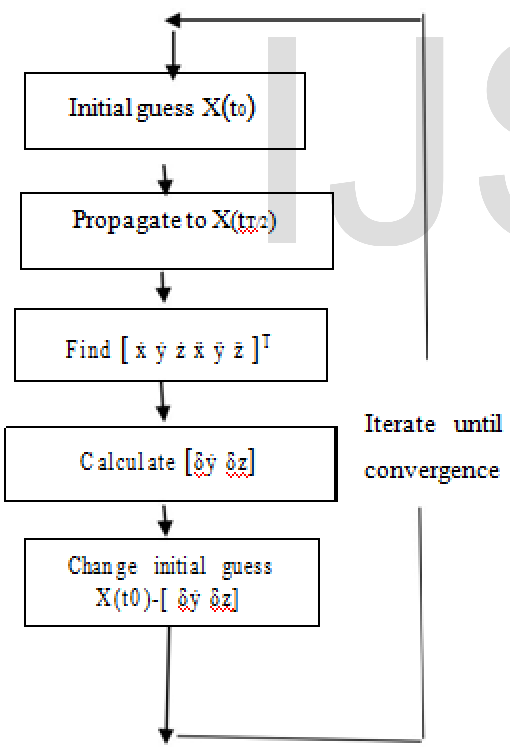
3.3 Computational Algorithm for differential correction method

The theory has been implemented to build a halo orbit around L1. An initial state was found

$X(t_0)$ = [x(fix) 0 -0.0123 0 0.01285 0 $]^T$ for sun-earth system

The initial state was then propagated forward in time for one period using a numerical integrator, in this case ODE113 in MALAB. The A-matrix to propagate the state transition matrix was generated using the function Jacobian in
MATLAB.

The next step was to find the true halo orbit based on the initial state.differential correction method was employed to find the true halo orbit .the initial state was propagated until it crossed the y=0 plane, which can be easily found by setting a constraint in the options for the ODE113 function.The deviation in **z** position and **y** velocity was found from the equations of motion and was then propagated using a special state transition matrix specified in the background section.the deviations were applied to the initial state and iterated until convergence.the below figure show the flow chart of the algorithm to obtain the true periodic halo orbit



Flow chart

3.4 Results and Discussion of Differential correction method

The initial state has a small error which manifests itself by not completely closing. Differential corrections method was employed ton find the true period halo orbit which can be seen in below figures. The orbit closes well and encircles the L1 Lagrange point in the and Sun –Earth system.the designed orbit is of 40,000km and it has time period of 170days.

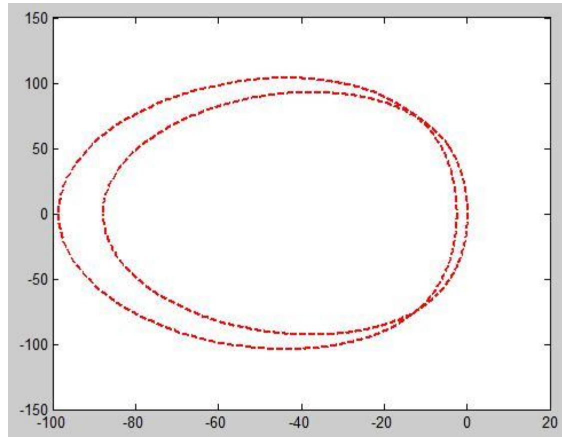


Fig 4: Halo orbit around L1 point for sun-earth system in x-y plane

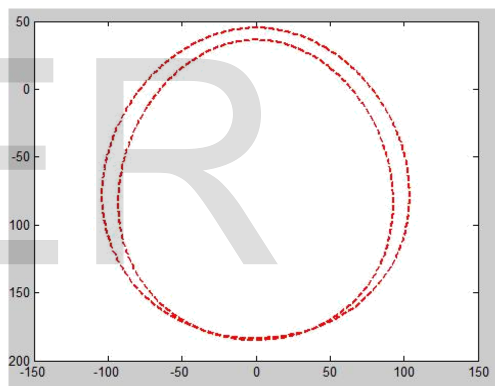


Fig 5: Halo orbit around L1 point for sun-earth system in x-z plane

4 Conclusions

The Halo orbit around L1 point for sun-earth system designed by determining initial conditions through differential correction method. Design of halo orbit is done using ODE113 solver of MATLAB, Period and amplitudes of halo orbit is calculated.

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